Benha University
Faculty of Engineering- Shoubra
Eng. Mathematics & Physics Department
Preparatory Year

Final Term Exam
Date: 17 – 5 – 2014
Course: Mathematics 1 – B
Duration: **3** hours

- **Answer All Questions** **The Exam consists of one page**
- **No. of questions: 4** **Total Mark: 100**

Question 1

(a) (i) Complete: The eigenvalues of a symmetric matrix of real numbers are………. **4**

(ii) State the types of solutions of a linear system AX = B.

(b) If $A = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 2 & 0 \\ 3 & 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$. Find, if possible, A + B, A.A`, A.B, |A|, B| **6**

(c) If $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$. Find (i)The eigenvalues and eigenvectors of A

(ii)The eigenvalues of $f(A) = A^5 - 2A$ (iii) $f(A) = A^n$ **15**

Question 2

(a)Solve the linear system: y – z = –3, x + 2y + 2z = 3, x + 3y + z = 0, 2x + y –z = –1 **5**

(b)Find S , S_{10} from: (i) $\sum_{r=1}^{n}(r+1)(r+2)$ (ii) $\sum_{r=1}^{n}\frac{1}{r^2+3r+2}$ (iii) $\sum_{r=1}^{n}\frac{2r+3}{[(r+1)(r+2)]^2}$ **12**

(c)Using the mathematical induction, prove that:

(i) $\frac{1}{1x3}+\frac{1}{3x5}+\frac{1}{5x7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$ (ii)If $y=\frac{1}{1+2x}$, then $y^{(n)}=\frac{(-2)^n.n!}{(1+2x)^{n+1}}$ **8**

Question 3

(a) Write down the equation for a rotation of axis through an angle $\pi/4$. Hence prove that the curve $2xy = a^2$ cane be transformed to $x^2 - y^2 = a^2$. **5**

(b) Find the equation of the parabola with focus at $(3,-4)$ and the directrix is $6x - 7y + 5 = 0$. **8**

(c) Find the equation of the ellipse whose foci $(\pm 4,\ 0)$ and its eccentricity is $1/3$. **8**

(d) Find the equation of the circle with center at $(6,6)$ and touch the circle $x^2 + y^2 = 32$. **5**

Question 4

(a) Find the equation of the two tangents of the hyperbola $9x^2 - 4y^2 = 36$ drawn from the point$(0,9)$. Find the angle between them. **8**

(b) Prove that the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ touch the axis. Hence find the equation of the circle which touches the axis at a distance 4 from the origin. **8**

(c) What conic the equation $4x^2 - 9y^2 - 16x + 54y - 101 = 0$ represent ? find its foci and equation of its directrix. **8**

Good Luck

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